29 January 1962   L-2044

Dr. I. J. Good
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Dear Dr. Good:

I have just come across references, in your comments on C. A. B. Smith's
article, "Consistency in Statistical Inference and Decision," to two
recent papers by you which I am extremely anxious to obtain. One is in
the Proceedings of the International Congress for Logic, Methodology and
Philosophy of Science, 1960; the other, in the Fourth London Symposium on
Information Theory. Since neither of these seems to be available yet in
this country, I would appreciate very much receiving reprints of these
papers, if any are available. In fact, since I am planning to present a
Ph.D. thesis on March 1st (to Harvard) on a subject to which I suspect
these papers have the highest pertinence, I would be happy to pay the cost
of having them sent airmail.

I am enclosing a copy of a recent paper of mine which should make clear
the reasons for my interest in your work. Since writing it -- unfortunately
not before -- I came across your 1952 article, "Rational Decisions," which
led me to track down everything I could find of your earlier and subsequent
writings. What leaped out at me in "Rational Decisions" was your clear
discussion (p. 114) of the same decision rule analyzed in the paper I had
just published, a rule which I had seen previously described only by
Hurwicz and by Hodges and Lehman (in papers cited in my enclosed article).
The "wooliness," "vagueness" or "lack of definition" of subjective probabi-
lities which you suggest as the motivation for this rule seems closely
related to the "ambiguity" which I discuss; and your notion (like that
of Koopman) of subjective probabilities as being bounded by inequalities
expresses the notion of "ambiguity" quite precisely. Although your dis-
cussion is almost always in terms of relations between beliefs, whereas
my argument, following that of Savage and Ramsey, focuses upon relations
between actions, I think the relationship between the two approaches is
clear. Those situations in which I assert (in the paper) that reasonable
people often act in violation of the Savage axioms are undoubtedly those
in which their probability judgments are in the form of inequalities,
with significant intervals between what Koopman calls their "upper and
lower probabilities."

The enclosed paper really represents my thinking as of 1957-1958.
Since publishing it at last, this fall, I have been developing the
subject further as a doctoral thesis. Whereas my earlier approach
had been entirely from the point of view of decision theory, I have
lately been particularly stimulated by the work on intuitive probability
by Keynes, Koopman, and yourself.

My article does confront directly a problem which you have not discussed
explicitly, one on which I would be most interested to have your comments.
The "Type II minimaxing" you describe in several contexts does have the
implication that it would lead to violations of the Savage postulates,
specifically his Postulate 2, the Sure-Thing Principle. The question I
consider is whether Type II minimaxing can, in view of this consequence,
still be regarded as "reasonable." I argue that it can. I suspect,
from your comments on "taking inequalities seriously" that you would
agree. Still, my position remains a lonely one in the published litera-
ture; I am not aware of any other published statement even suggesting
that it might be acceptable in any circumstances to violate Savage's
Sure-Thing Principle, let alone identifying any specific circumstances
in which this might be reasonable. Your own recent comments on the
importance of inequalities and the reasonableness of the behavior
described by Smith encourages me to believe that you are prepared to
part company with the Sure-Thing Principle in situations where intuitive .
probabilities are "non-appraisable."

The issue seems to me to be this. Suppose that, as you put it, the
initial probabilities are "'known' only to lie in wide intervals"; the
question arises, How may one reasonably act on the basis of such vague
estimates? Certain decision rules with respect to these interval esti-
mates of probabilities would not lead to violation of the Savage axioms:
for example, the rule of maximizing expected utility with respect to the
probabilities at the mid-point of each interval. Other decision rules,
such as the one you label "Type II minimaxing," would lead to violations
of the Savage axioms. Savage, in effect, argues that the latter rules
are to be rejected as leading to unreasonable behavior. I do not agree;
nor, I take it, do you. Smith's position on this is not quite clear to
me; if I understand his implied decision rule correctly (and I am not
sure that I do), he suggests that one pick a probability estimate randomly
from the interval of reasonable estimates. This would lead to violations
of Savage's Postulate 1, but not, I suspect, to closely related sets of
stochastic axioms such as those proposed by Luce.

It happens that I once had a brief opportunity to present my views
to you. You probably don't remember the occasion, but it was an
historic day for me, since I had just an hour earlier confronted
Savage with my examples and won from him (to the surprise of both of
us) the admission that he would wish in these circumstances to violate
his own postulates. (He recently commented that he still feels the
intuitive "pull" of the example, though he reserves judgment as to the
implications of this for the axioms.) As I remember, you were just
preparing to leave Chicago at that moment, and Savage brought me over
to the Faculty Club to test your reactions to the same questions. To
the best of my recollection (this was, I believe, about February 1958),
your immediate responses did not violate the axioms, though this
scarcely dampened my spirits on that particular day. I would be
delighted to find, of course, that you had changed your mind.

                                        Very truly yours,

                                        Daniel Ellsberg
                                        Economics Department

DE/ss
Enclosure

     P-2173, Risk, Ambiguity,
     and the Savage Axioms,
     dtd. 8/25/61

     bcc: Reports
           B. Haydon